

B.Tech. (ME) 2nd Semester G-Scheme

Examination, May-2019

MATH-II

Paper-BSC-MATH-102-G

(Multivariables Calculus, Differential Equations and
Complex Analysis)

Time allowed : 3 hours]

[Maximum marks : 75

*Note : Question No. 1 is compulsory. Attempt five questions
in total by selecting one question from each unit.
All questions carry equal marks.*

1. (a) Evaluate $\int_1^2 \left[\int_3^4 (xy + e^y) dx \right] dy$.

(b) Solve $(2x \cos y + 3x^2y) dx +$
 $(x^3 - x^2 \sin y - y) dy = 0$

(c) Define Mobius transformation, and when it is called univalent.

(d) prove that $\sinh z$ is analytic function.

(e) If n is an integer s.t. $n \neq -1$ and C is circle
 $|z - a| = r$. Then prove $\oint_C (z - a)^n dz = 0$

(f) Evaluate $\oint_C (x^2 - y^2 + 2ixy)$, where C is the
contour $|z| = 1$. 6×2.5=15

Unit-I

2. (a) Evaluate $\iint \frac{(x-y)^2}{x^2+y^2} dx dy$, over the circle $x^2 + y^2 \leq 1$. 7.5

- (b) By changing the order evaluate the integral

$$\int_0^3 \int_1^{\sqrt{4-y}} (x+y) dx dy. \quad 7.5$$

3. Verify Stoke's Theorem for $\vec{f} = x^2\hat{i} + xy\hat{j}$, integrated around the square in the plane $z=0$, whose sides are along the lines $x = 0$, $x = a$, $y = 0$ and $y = a$. 15

Unit-II

4. (a) Solve the equation $\frac{d^2y}{dx^2} + y \operatorname{cosec} x$ by using method of variation of parameters. 7.5

- (b) Solve Cauchy-Euler equation:

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin (\log x). \quad 7.5$$

5. (a) Express the polynomial x^3+2x^2-x-3 in terms of Legendre's polynomials. 7

- (b) Find the power series solution about $x=0$, of $(1-x^2) y'' - 2xy' + 2y = 0$. 8

Unit-III

6. State and prove necessary and sufficient conditions for $f(z)$ to be analytic. 15
7. (a) Show that the function $u=e^{-2xy} \sin (x^2-y^2)$ is harmonic. Find the conjugate function v and express $u + iv$ as an analytic function of z . 8
- (b) Determine the analytic function whose real part is $(e^x \cos y - e^y \sin y)$. 7

Unit-IV

8. (a) Expand $\frac{e^{2z}}{(z-1)^3}$ in Laurent's series about its singularity. 7.5
- (b) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1,2,3$ and ∞ , also determine their sum. 7.5
9. (a) Verify Cauchy's integral theorem by integrating e^z along the boundary of the triangle with vertices at the points $1+i, -1+i$ and $-1-i$. 8
- (b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4(z+5)} dz$, where C is the circle $|z|=2$. 7