B.Tech. (ME) 2nd Semester G-Scheme

Examination, May-2019

MATH-II

Paper-BSC-MATH-102-G

(Multivariables Calculus, Differential Equations and Complex Analysis)

Time allowed : 3 hours] *[Maximum marks : 75*

Note : ***Question No. 1 is compulsory.*** *Attempt* ***five*** *questions in total by selecting* ***one*** *question from each unit. All questions carry equal marks.*

1. (a) Evaluate $\int_1^2 \left[\int_3^4 \left(xy + e^y \right) dx \right] dy$.

(b) Solve $(2x \cos y + 3x^2y)\, dx + (x^3 - x^2 \sin y - y)\, dy = 0$

(c) Define Mobius transformation, and when it is called univalent.

(d) prove that sinh z is analytic function.

(e) If n is an integer s.t. $n \neq -1$ and C is circle $|z-a| = r$. Then prove $\oint_C (z-a)^n dz = 0$

(f) Evaluate $\oint_C (x^2 - y^2 + 2ixy)$, where C is the contour $|z| = 1$. 6×2.5=15

Unit-I

2. (a) Evaluate $\iint \frac{(x-y)^2}{x^2+y^2}\, dx\, dy$, over the circle $x^2 + y^2 \leq 1$. 7.5

   (b) By changing the order evaluate the integral

   $$\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\, dx\, dy.$$ 7.5

3. Verify Stoke's Theorem for $\bar{f} = x^2\hat{i} + xy\hat{j}$, integrated around the square in the plane z=0, whose sides are along the lines x = 0, x = a, y = 0 and y = a. 15

Unit-II

4. (a) Solve the equation $\frac{d^2y}{dx^2} + y \operatorname{cosec} x$ by using method of variation of parameters. 7.5

   (b) Solve Cauchy-Euler equation:

   $$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x).$$ 7.5

5. (a) Express the polynomial x^3+2x^2-x-3 in terms of Legendre's polynomials. 7

   (b) Find the power series solution about x=0, of $(1-x^2)\, y''-2xy'+2y=0$. 8

Unit-III

6. State and prove necessary and sufficient conditions for f(z) to be analytic. 15

7. (a) Show that the function $u=e^{-2xy}\sin(x^2-y^2)$ is harmonic. Find the conjugate function v and express u + iv as an analytic function of z. 8

   (b) Determine the analytic function whose real part is $(e^x x\cos y - e^x y\sin y)$. 7

Unit-IV

8. (a) Expand $\frac{e^{2z}}{(z-1)^3}$ in Laurent's series about its singularity. 7.5

   (b) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1,2,3$ and ∞, also determine their sum. 7.5

9. (a) Verify Cauchy's integral theorem by integrating e^{iz} along the boundary of the triangle with vertices at the points $1+i$, $-1+i$ and $-1-i$. 8

   (b) Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4(z+5)}\,dz$, where C is the circle $|z|=2$. 7